3 November 2020

Exam Solid Mechanics

1) In 1913, Von Mises proposed the following scalar-valued representation of a stress tensor:

$$\sigma_v = \sqrt{3/2\, \underline{\underline{\sigma}}' : \underline{\underline{\sigma}}'}$$

Where, as usual

$$\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} - (1/3 \operatorname{tr} \underline{\underline{\sigma}}) \underline{\underline{I}} \quad \text{is the stress deviator}$$

(a) Compute σ_v for uniaxial tension at a stress σ, as well as for hydrostatic compression with magnitude p.

<u>Uniaxial tension</u>

For uniaxial tension the given stress is $\underline{\underline{\sigma}} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underline{e}_i \otimes$

So I can calculate the ~~trac~~ stress deviator

$$\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} - (1/3 \operatorname{tr} \underline{\underline{\sigma}}) \cdot \underline{\underline{I}} = \left(\sigma \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - 1/3 \cdot \sigma \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \underline{e}_i \otimes$$

$$= 1/3\, \sigma \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j$$

Now I'll calculate $\underline{\underline{\sigma}}' : \underline{\underline{\sigma}}'$. Knowing that $\underline{\underline{A}} : \underline{\underline{B}} = A_{ij} B_{ji}$

$$\underline{\underline{\sigma}}' : \underline{\underline{\sigma}}' = (1/3\, \sigma)^2 \left(2^2 + (-1)^2 + (-1)^2 \right) = 1/9\, \sigma^2 \cdot 6 = 2/3\,\sigma^2$$

$$\sigma_v = \sqrt{3/2\, \underline{\underline{\sigma}} : \underline{\underline{\sigma}}} = \sqrt{3/2 \cdot 2/3 \cdot \sigma^2} = |\sigma|$$

Hydrostatic compression

For hydrostatic compression the given stress is $\underline{\underline{\sigma}} = p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} e_i \otimes e_j$

~~Solution~~

So $\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} - \left(\tfrac{1}{3} \operatorname{tr} \underline{\underline{\sigma}}\right) \cdot \underline{\underline{I}} = p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \tfrac{1}{3} \cdot 3p \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$

this means that $\sigma_v = \sqrt{\tfrac{3}{2} \cdot 0 : 0} = 0$

There is no Von Mises stress

(b) Express σ_v in terms of shear stress τ in the case of pure shear defined by: $\sigma_{12} = \sigma_{21} = \tau$, otherwise $\sigma_{ij} = 0$

For this we know that the stress tensor ~~is~~ is given by

$$\underline{\underline{\sigma}} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

Now $\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} - \left(\tfrac{1}{3} \operatorname{tr} \underline{\underline{\sigma}}\right) \cdot \underline{\underline{I}} = \left(\begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \tfrac{1}{3} \cdot 0 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \underline{e_i} \otimes \underline{e_j}$

$$= \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

$$\underline{\underline{\sigma}}' : \underline{\underline{\sigma}}' = \tau^2 + \tau^2 = 2\tau^2$$

So $\sigma_v = \sqrt{\tfrac{3}{2} \cdot 2\tau^2} = \sqrt{3} \, |\tau|$

(c) According to Mohr's circle discussed in Chapter 2.6, pure shear can be regarded as a combination of tension and compression in two mutually orthogonal directions. Does this also hold for the corresponding Von Mises stresses from (a)?

~~In (a) we have the situation of uniaxial tension and hydrostatic compression.~~

The Von Mises stress is a scalar representation of stress, while a stress tensor has a 2-vector basis. So the Von Mises stress will tell ~~saup~~ something about the stress, but not ~~everyth...~~ give a lot of information about the directions. For uniaxial tension there is only 1 direction, so the Von Mises stress is equal to the magnitude of the stress. For hydrostatic pressure this is completely differend. Because the directions cancel out, the Von Mises stress becomes 0, while the stress isn't. So here it doesn't hold.

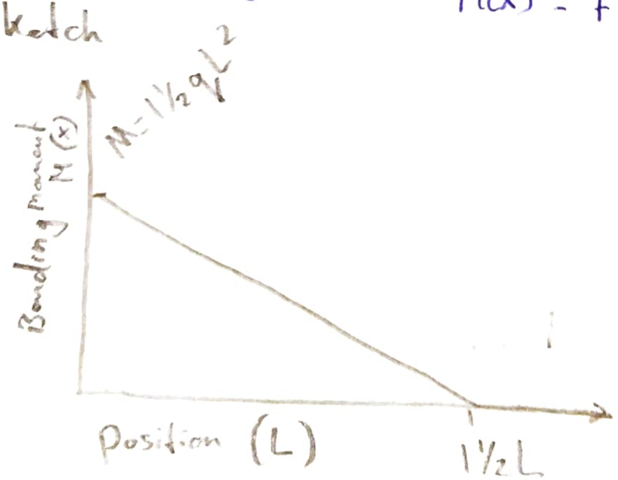
2)

(a) Determine (and Sketch) the distribution of the bending moment for both cases and discuss the salient differences.

I begin with situation (b), because this problem doesn't use super-position and is thus simpler. The situation is here that there is a beam with a force on a point on the beam at $1\frac{1}{2}L$
$F = q \cdot L$

Because the force is at one point the distribution of bending moment is linear, with $M = 0$ at the point where the beam is loaded. Where ~~ø~~ the beam is not loaded or clamped the bending moment is zero i.e. there is no bending moment for $1\frac{1}{2}L$ to $2L$. For the part $L = 0$ to $L = 1\frac{1}{2}$ ~~the~~ the there is a bending moment $M(x) = F(1\frac{1}{2}L - x) = qL(1\frac{1}{2}L - x)$

Sketch



Now for situation (a). Here we require Superposition to determine the bending moment distribution.

This problem consists out of 2 parts.

A part that only feels the force at the tip because of the force balance and a part of a continuous force distribution.

Because of the force distribution at the end at that part there will be a double dependence on position.

If it would continue ~~ofer~~ over the whole beam we would get a dependence like $M(x) = \frac{qL(2L-x)^2}{4L} = \frac{q(2L-x)^2}{4}$

So this will hold for the part between $1L$ and $2L$

So at $1L$ there will be a bending moment of:

$$M(L) = \frac{qL \cdot L^2}{4L} = \frac{1}{4} q L^2$$

For the part between 0 and L we'll have a Superposition. If the beam was only loaded at L it would give us the bending moment $M(x) = qL(L-x)$

but because of the superposition we'll have ~~M(x)=~~

$$M(x) = qL(L-x) + \frac{1}{4}qL^2 = qL(L - x + \frac{1}{4}L) = qL(1\frac{1}{4}L - x)$$

Because this is a lenghtly story I'll summarize it.

$$M(x) = \begin{cases} qL\left(1\tfrac{1}{4}L - x\right) & 0 \leq x \leq L \\ \dfrac{q(2L - x)^2}{4} & L < x \leq 2L \end{cases}$$

Sketch

~~(b) Use superposition and the third forget-me-not (only!), to show that the end deflection for configuration (a) is given by~~

~~$$w_a = \frac{41}{24}\frac{qL^4}{EI}$$~~

The salient differences between (a) and (b) are that in (b) the distribution is linear and in (a) it is not ~~Am~~. And in (a) the bending moment starts farther away from where the beam is clamped, I think that this also leads to a higher total bending moment.

(b) Use superposition ~~to show that the~~ and the third forget-me-not (only!), to show that the end deflection for configuration (a) is given by

$$W_1 = \frac{41}{24} \frac{qL^4}{EI}$$

Third forget-me-not ~~(1b)~~ $w(L) = \frac{qL^4}{8EI}$

This is a superposition problem consisting out of 2 parts

A part that only experiences a force at the end tip and a part that is continuously loaded.

~~First the deflection due to the continuous loading~~ Bending

~~I call it W_{a2}~~

~~Here I use the forget me not~~.

I am not able to do this using only the 3rd forget-me-not due to the given time, but I hope my solution makes sense

There are two parts: 1a and 1b

For 1a $W_{1a}(L) = \frac{FL^3}{3EI} = \frac{qL^4}{3EI}$

2 1b $W_{1b}(L) = \theta_{1a}(L) \cdot L + \frac{qL^4}{8EI}$

$$= \frac{qL \cdot L^2}{2EI} \cdot L + \frac{qL^4}{8EI} = \frac{4qL^4}{8EI} + \frac{qL^4}{8EI} = \frac{5qL^4}{8EI}$$

Total deflection.

$$W(L) = W_{1A} + W_{1b} = \frac{qL^4}{3EI} + \frac{5qL^4}{8EI}$$

$$= \frac{8qL^4}{24EI} + \frac{\cancel{5}\,15qL^4}{24\cancel{8}EI} = \frac{23qL^4}{24EI}$$

This ~~4~~ is different than the deflection I should get.

c) Express the end deflection for configuration (b) in terms of q, L and EI.

I choose ~~the~~ to use the forget me nots to solve this problem.

~~Total tt~~ $w_1(L) = \frac{FL^3}{3EI}$ $\qquad \theta(L) = \frac{FL^2}{2EI}$

$$w(1\tfrac{1}{2}L) = \frac{F\,(1\frac{1}{2}L)^3}{3EI} = \frac{qL \cdot 3\frac{3}{8}\, qL^4}{3EI} = \frac{9\cancel{8}\, q L^4}{8\cancel{q}\, EI}$$

There is also a part that deflects, but doesn't bend.

This deflection is given by $w_2 = \theta(1\frac{1}{2}L)\cdot \frac{1}{2}L = \frac{qL\,(1\frac{1}{2}L)^2}{2EI}\cdot \frac{1}{2}L$

$$= \frac{9}{16}\cdot\frac{L^4}{2EI}$$

So the total deflection is given by $w(L) = w_1 + w_2 =$

$$\frac{9qL^4}{8EI} + \frac{9qL^4}{16EI} = \frac{27qL^4}{16EI}$$

This answers differs from $\frac{41}{24}\cdot\frac{qL^4}{EI}$

In the sense that $\frac{41}{24}$ is slighty larger than $\frac{27}{16}$, but the difference is really small. ~~So the~~ $\frac{41}{24} \approx 1{,}708$, $\frac{27}{16} \approx 1{,}688$

So the accuracy is pretty good. It's close.

1) Consider a homogeneous plate subject to tension in the $\underline{e_1}$ direction. The plate is made of a single crystal, but its orientation is unknown. The elastic properties of the crystal are isotropic.

(a) First assume a uniaxial stress state. Determine the most likely slip plane and slip direction, i.e. where the resolved shear stress is maximum.

Here we have uniaxial tension and we want to find a maximum shear stress. This is planar stress. The maximum shear stress will probably be found at a 45° angle relative to the $\underline{e_1}$ axis. This holds for the $\underline{e_1}-\underline{e_2}$ plane, but also for the $\underline{e_1}-\underline{e_3}$ plane. So I think this will be the most likely slip planes.

(b) Repeat the analysis in case the plate is very wide in the $\underline{e_2}$-direction, so that it is in a state of plane strain tension (with $\varepsilon_{22}=0$)

The plain is very wide in the e_2 direction, so we can consider it to be constrained in the $\underline{e_2}$ direction.
~~the~~ We have plane strain tension.

In this case we get that $\sigma_{22} = \nu\sigma$ and $\sigma_{11} = \sigma$ stays the same. All other components must be zero due to the boundary conditions.

This gives us a stress tensor $[\sigma_{ij}] = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma v & 0 \\ 0 & 0 & 0 \end{bmatrix}$

We want to know stuff about shear stresses, so Mohrs circle applies. Our principal stresses are $\sigma_{11} = \sigma$, $\sigma_{22} = \sigma v$ and $\sigma_{33} = 0$. For the largest shear stress $\sigma > \sigma v > 0$.

The largest shear stress is found in the $\underline{e_1} - \underline{e_3}$ plane, So this is the most likely slip plane and slip direction.